Abstract—This paper presents a robust model predictive current controller with a disturbance observer (DO-MPC) for three-phase voltage source PWM rectifier. The new algorithm is operated with constant switching frequency (CF-MPC). In order to minimize instantaneous d- and q-axes current errors in every sampling period, CF-MPC is implemented by selecting appropriate voltage vector sequence and calculating duty cycles. The fundamental of this algorithm is discussed and the instantaneous variation rates of d- and q-axes currents are deduced when each converter voltage vector is applied in six different sectors. A Luenberger observer is constructed for parameter mismatch and model uncertainty which affect the performance of the MPC. The gains of the disturbance observer are determined by root-locus analysis. Moreover, the stability of the disturbance observer is analyzed when there are errors in the inductor filter parameter. The proposed method has an inherent rapid dynamic response as a result of the MPC controller, as well as robust control performance with respect to the disturbance due to use of the combined observation algorithm. Simulation and experimental results on a 1.1 kW VSR are conducted to validate the effectiveness of the proposed solution.

Index Terms—Rectifier, model predictive current controller, disturbance observer, stability analysis, root-locus analysis, parameter errors.

I. INTRODUCTION

THREE-PHASE voltage source PWM rectifiers (VSRs) have been increasingly employed in various industrial applications, such as adjustable speed drives, uninterruptible power supplies, distributed and renewable energy generation, etc. [1]-[4]. It has the advantages of low line current distortions with unity power factor operation and constant dc-link voltage with a small output filter capacitor.

In the recent works, various control strategies have been proposed for the VSR. As one of the most popular methods, voltage-oriented control (VOC) [5]-[7] is of indirect active and reactive power control. Also it ensures high static and dynamic performance of power control via internal current control loops indirectly. For this control algorithm, unity power factor operation can be achieved when the line current vector of the power line for VSR is aligned with the phase voltage vector. Due to the nonlinear feature of the converter, a nonlinear control technique called direct power control (DPC) has been presented based on instantaneous active and reactive power control loops [8]-[11]. In the conventional DPC, there are no internal current control loops and no PWM modulator block as the converter switching states are selected from a switching table. This algorithm selects an appropriate voltage vector sequence and calculates duty cycles in every sampling period in order to minimize instantaneous active and reactive power errors. The lack of linear controllers and modulator makes the system respond very fast in transients.

Nowadays, there is a strong trend toward fully digital control of PWM converters based on model predictive control (MPC) strategies [12]-[13]. MPC is regarded as a class of computer control algorithms utilizing an explicit process model to predict the future response of a plant. Due to minimization of cost function which defines behavior of the system, most effective voltage vector is selected for next sampling period and the algorithm is recalculated every sampling period. In the past, MPC has only been applied to slow systems considering the large number of complex computations required to solve the optimization problem at each sampling instant [14]. The growth of the available computational ability, together with some recent research works, has shown the potential of MPC strategies in power electronics applications [15]-[20].

However, the MPC with variable switching frequency has the drawbacks of high sampling frequency requirement, causing much more switching losses and demanding faster computation speed which imposes a new burden on the controller and increases the cost. Furthermore, the existing MPC-based control techniques in many references are unable to explicitly incorporate plant model uncertainty [12], [21] and
VSR is designed. Section IV shows the simulation results on a predictive current controller based on a disturbance observer of stability of the observer is discussed. Then, the robust model predictive current controller for parameters variation and noise interference. This paper is organized as follows. Section II depicts the VSR model and CF-MPC principle. A constant switching-frequency MPC strategy (CF-MPC) with a modulator is developed, allowing easy electromagnetic interference (EMI) filter design and protection of semiconductor power components [29]. When combined with modulation technique, the presented MPC strategy combined with sliding mode control is proposed in [30]. The proposed disturbance observer allows for implementation of robust MPC current controller for parameters variation and noise interference.

In this paper, an innovative robust model predictive current controller based on a disturbance observer (DO-MPC) suitable for use in VSR is presented. A constant switching-frequency MPC strategy (CF-MPC) with a modulator is developed, allowing easy electromagnetic interference (EMI) filter design and protection of semiconductor power components [29]. When combined with modulation technique, the presented control scheme is supposed to provide the lowest distortion and current ripple [30]. The proposed disturbance observer algorithm is referred to the observer proposed in [31] and is constructed using grid voltage, input voltage and current of rectifier as the input. The observer is applied to estimate model uncertainties, unknown external disturbances, and time-varying parameters by minimizing the difference between the measured current and the modeled current. Therefore, the disturbance observer allows for implementation of robust MPC current controller for parameters variation and noise interference.

This paper is organized as follows. Section II depicts the VSR model with and without the disturbances observer in the presence of parameter mismatch, and analyzes the effect of disturbance observer on the stability of VSR system. In Section V, experimental results on a 1.1 kW VSR test bench is provided. Finally, the conclusions are drawn in Section VI.

II. VSR MODEL AND CF-MPC PRINCIPLE

A. VSR Model

A common topology of a VSR is depicted in Fig. 1. The rectifier is connected to the grid through inductance \( L_g \) and its equivalent serial resistance \( R_g \).

![Fig. 1. Topological structure of main circuit of a three-phase boost-type PWM rectifier.](Image)

where \( u_{ga}, u_{gb}, \) and \( u_{gc} \) are the source voltages; \( i_{ga}, i_{gb} \) and \( i_{gc} \) are the input currents of rectifier; \( u_{ca}, u_{cb}, \) and \( u_{cc} \) are the rectifier input voltage; \( u_{dc} \) is the output dc voltage.

The mathematical model of VSR in abc frame can be expressed by

\[
\begin{align*}
L_g \frac{di_{ga}}{dt} &= \begin{bmatrix} -R_g & 0 & 0 \ 0 & -R_g & 0 \ 0 & 0 & -R_g \end{bmatrix} i_d + \begin{bmatrix} u_{ga} - u_{ca} \ u_{gb} - u_{cb} \ u_{gc} - u_{cc} \end{bmatrix} \\
L_g \frac{di_{gb}}{dt} &= 0 \\
L_g \frac{di_{gc}}{dt} &= 0 \\
C \frac{du_{dc}}{dt} &= S_a S_b S_c -1 \begin{bmatrix} u_{ca} \ u_{cb} \ u_{cc} \end{bmatrix}
\end{align*}
\]

where \( S_a, S_b, \) and \( S_c \) stand for the switching state of each phase leg in Fig. 1, respectively. \( S_k = 1 \) (k = a, b, c) means that the upper switch of phase k is turned on, and the lower switch is turned off while \( S_k = 0 \) denotes the opposite meaning.

The aforementioned equations can be transformed into two-phase stationary (\( \alpha-\beta \)) frame by Clarke’s matrix

\[
T = \frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ -1/2 & 3/2 & -3/2 \\ -1/2 & -3/2 & 3/2 \end{bmatrix}
\]

\[ T \]
Based upon Clarke’s transformation, the dynamic model of VSR can be described as

\[
\begin{bmatrix}
u_{gd} \\ u_{gq}
\end{bmatrix} = L_g \begin{bmatrix}
\frac{di_{gd}}{dt} \\ \frac{di_{gq}}{dt}
\end{bmatrix} + \begin{bmatrix}
R_g & 0 \\ 0 & R_g
\end{bmatrix} \begin{bmatrix}
i_{gd} \\ i_{gq}
\end{bmatrix} + \begin{bmatrix}
u_{ca} \\ u_{cq}
\end{bmatrix}
\] (4)

In Park’s d-q frame that rotates synchronously with the grid voltage angular speed, the voltage equation of VSR can be reasonably represented by the following equations:

\[
\begin{bmatrix}
u_{gd} \\ u_{gq}
\end{bmatrix} = L_g \begin{bmatrix}
\frac{di_{gd}}{dt} \\ \frac{di_{gq}}{dt}
\end{bmatrix} + \begin{bmatrix}
R_g - \omega_s L_g & \omega_s L_g \\ \omega_s L_g & R_g - \omega_s L_g
\end{bmatrix} \begin{bmatrix}
i_{gd} \\ i_{gq}
\end{bmatrix} + \begin{bmatrix}
u_{cd} \\ u_{cq}
\end{bmatrix}
\] (5)

where \(u_{cd}, u_{cq}, i_{gd}, i_{gq}\) are the d- and q-axes rectifier’s input voltages and currents; \(u_{gd}, u_{gq}\) are the d- and q- axes components of the grid voltage; \(\omega_g\) is the voltage angular frequency.

### B. Principle of CF-MPC on VSR

An important issue in power conversion is the efficiency of the power converter due to power loss in long-term operations. Switching losses are especially relevant as they are produced by the commutation of the power devices [32]. Therefore, switching losses are a direct result of the control of a modulation method. In this section, it will be focused on the constant frequency control method of MPC in which the outputs of the controller firstly pass through the modulator and then the modulator provides the switch positions. The application of modulator requires low sampling frequency. For variable switching frequency control scheme, the frequency of disturbance caused by power device is not definite and current harmonics may exist on different frequency band which increases the difficulty to design the filter. However, as to the constant switching frequency algorithm, the harmonics will focus on constant switching frequency and corresponding filter can be designed according to the constant switching frequency which is convenient for EMI filter design.

Based on (5), the instantaneous current variations can be expressed with respective d- and q-axes components as

\[
\begin{bmatrix}
\frac{di_{gd}}{dt} \\ \frac{di_{gq}}{dt}
\end{bmatrix} = \frac{1}{L_g} \begin{bmatrix}
-R_g & \omega_s L_g \\ -\omega_s L_g & R_g - \omega_s L_g
\end{bmatrix} \begin{bmatrix}
i_{gd} \\ i_{gq}
\end{bmatrix} + \frac{1}{L_g} \begin{bmatrix}
u_{gd} - u_{cd} \\ u_{gq} - u_{cq}
\end{bmatrix}
\] (6)

It can be implied from (6) that the change rate of the instantaneous current is affected by system parameters, current, grid voltage and together with rectifier input voltage.

As shown in Fig. 1, the three-phase rectifier voltages can be represented by eight voltage vectors, i.e., six active vectors and two zero vectors. Fig. 2 presents the eight voltage vectors denoted as \(V_0\) - \(V_7\). The \(\alpha\) - and \(\beta\)-axes components of each converter voltage vector are given in Table I.

![Eight voltage vectors in the \(\alpha\)-\(\beta\) reference frame.](image)

<table>
<thead>
<tr>
<th>Voltage vector</th>
<th>(v_{\alpha})</th>
<th>(v_{\beta})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_1) (001)</td>
<td>(2v_{dc}/3)</td>
<td>0</td>
</tr>
<tr>
<td>(V_2) (010)</td>
<td>(-v_{dc}/3)</td>
<td>(\sqrt{3}v_{dc}/3)</td>
</tr>
<tr>
<td>(V_3) (011)</td>
<td>(v_{dc}/3)</td>
<td>(\sqrt{3}v_{dc}/3)</td>
</tr>
<tr>
<td>(V_4) (100)</td>
<td>(-v_{dc}/3)</td>
<td>(-\sqrt{3}v_{dc}/3)</td>
</tr>
<tr>
<td>(V_5) (101)</td>
<td>(v_{dc}/3)</td>
<td>(-\sqrt{3}v_{dc}/3)</td>
</tr>
<tr>
<td>(V_6) (110)</td>
<td>(-2v_{dc}/3)</td>
<td>0</td>
</tr>
<tr>
<td>(V_7) (111)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

![Current error derivative vectors when the grid voltage vector is located in Sector (III).](image)

<table>
<thead>
<tr>
<th>Sector Number</th>
<th>(di_{\alpha}/dt)</th>
<th>(di_{\beta}/dt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
<td>(V_0, V_1, V_4)</td>
<td>(V_2, V_3)</td>
</tr>
<tr>
<td>(II)</td>
<td>(V_0, V_1, V_4)</td>
<td>(V_2, V_3)</td>
</tr>
<tr>
<td>(III)</td>
<td>(V_0, V_1, V_4)</td>
<td>(V_2, V_3)</td>
</tr>
<tr>
<td>(IV)</td>
<td>(V_0, V_1, V_4)</td>
<td>(V_2, V_3)</td>
</tr>
<tr>
<td>(V)</td>
<td>(V_0, V_1, V_4)</td>
<td>(V_2, V_3)</td>
</tr>
<tr>
<td>(VI)</td>
<td>(V_0, V_1, V_4)</td>
<td>(V_2, V_3)</td>
</tr>
</tbody>
</table>
As is shown in Fig. 3, \(di_d/dt\) becomes positive with voltage vector \(V_6, V_5, V_6\) or \(V_7\) applied throughout the whole Sector (III); \(V_1\) makes \(di_d/dt\) negative, except in the vicinity of \(\theta = 0^\circ\) while \(V_7\) makes \(di_d/dt\) negative, except in the vicinity of \(\theta = 60^\circ\). Therefore, \(di_d/dt\) is negative in most part of the sector when \(V_7\) and \(V_1\) is employed. Similarly, \(V_1, V_5, V_4\) keeps \(di_d/dt\) positive, while \(V_3, V_2, V_6\) maintains \(di_d/dt\) negative. In summary, the d-axis current is decreased in most part of the sector and the q-axis current is increased throughout the whole sector with vector \(V_7\) applied, while the d-axis current and the q-axis current throughout the whole sector are decreased with vector \(V_1\) adopted when the grid voltage is located in Sector (III). Likewise, the d- and q-axes currents variation rates produced by each converter voltage vector can be illustrated in Table II in all six sectors.

In this paper, three voltage vectors including two active vectors and one zero vector are applied within each control period \(T_s\). The selection of the two active voltage vectors in each sector is based on the location of grid voltage vector, to which the two nearest-located active vectors are used together with either \(V_6\) or \(V_7\) according to the rule of minimum commutation numbers. The switching states are obtained in Table III.

### Table III

<table>
<thead>
<tr>
<th>Sector Number</th>
<th>Switching Table for VSR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sector Number</td>
</tr>
<tr>
<td></td>
<td>(I) (II) (III) (IV) (V) (VI)</td>
</tr>
<tr>
<td>1</td>
<td>( V_1 ) ( V_2 ) ( V_1 ) ( V_2 ) ( V_0 ) ( V_2 )</td>
</tr>
<tr>
<td>2</td>
<td>( V_2 ) ( V_1 ) ( V_1 ) ( V_4 ) ( V_2 ) ( V_4 )</td>
</tr>
<tr>
<td>3</td>
<td>( V_1 ) ( V_3 ) ( V_2 ) ( V_3 ) ( V_5 ) ( V_3 )</td>
</tr>
</tbody>
</table>

The converter voltage in Park’s d-q transformation can be described as

\[
\begin{bmatrix}
u_d \\
u_q
\end{bmatrix} = \begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix} \begin{bmatrix}
u_{ca} \\
u_{cb}
\end{bmatrix}
\]

Substituting \(u_{ca}\) and \(u_{cb}\) of each converter input voltage vector into (6) yields the change rates of d- and q-axes currents as follows

\[
e_{id} = \frac{di_d}{dt} u_{ca}, \quad e_{iq} = \frac{di_q}{dt} u_{cb}
\]

In each sector, the effect of every converter voltage vector on the instantaneous changes of d- and q-axes currents can be given by (8). As a result, linear trajectories of d- and q-axes currents under an applied converter voltage vector with its duration time \(t_0\) can be computed as

\[
\Delta i_{d}\left(k+1\right) = i_{d}\left(k\right) - i_{d}\left(k\right) = e_{id} t_0
\]

\[
\Delta i_{q}\left(k+1\right) = i_{q}\left(k\right) - i_{q}\left(k\right) = e_{iq} t_0
\]

where \(i_{d}(k)\) and \(i_{q}(k)\) are the initial values of d- and q-axes currents at the beginning of voltage vector application, \(i_{d}(k+1)\) and \(i_{q}(k+1)\) are the instantaneous values of d- and q-axes currents at the end of the applied voltage vector, separately.

As can be seen in Fig.4, \(t_0, t_1\) and \(t_2\) are the duration time of three chosen voltage vectors.

According to Fig.4, the error of predicted d- and q-axes currents at the end of \(k\)th control period can be computed with their variations generated by the applied three voltage vectors

\[
E_d = t_0 \frac{d}{dt} i_{d}(k) - e_{id} t_0 - e_{id} t_1 - e_{id} t_2
\]

\[
E_q = t_0 \frac{d}{dt} i_{q}(k) - e_{iq} t_0 - e_{iq} t_1 - e_{iq} t_2
\]

where \(i_{d}(k)\) and \(i_{q}(k)\) are the predicted d- and q-axes currents at the end of the control period of \(T_s\). In such a condition, the duration time of zero vector is set to null, \(t_0 = T_s - t_1 - t_2\). Once the three voltage vectors are determined from Table III with the location information of grid voltage vector, their duration time in the next control period of \(T_s\) can be predicted by (13). However, it should be noted that the sum value of the predicted duration time \(t_1 + t_2\) may be bigger than \(T_s\).
whereas the duration time of two active vectors can be calculated as

\[ t_1' = \frac{t_1}{t_1 + t_2} T_s \]
\[ t_2' = \frac{t_2}{t_1 + t_2} T_s \] (14)

A modulator referred to the typical space vector modulation strategy is adopted in this paper and the switching pattern for the Sector (III) is depicted in Fig. 5.

Fig. 5. Switching pattern generation algorithm in Sector (III).

As shown in Fig. 5, each half of sampling period \( T_s/2 \) starts and ends with zero vectors. Therefore there will be two zero vectors per \( T_s/2 \) or four null vectors per \( T_s \), and the duration time of each null vector is \( t_o/4 \).

III. DISTURBANCE ESTIMATION

Luenberger observer adopts predictor-corrector structure where a model is used by the observer to predict system actual voltage reference for eliminating disturbances. The action ensures that the estimated disturbance term is added to noise. In order to provide a robust current control, the feed inaccuracy and eliminating the effect of disturbances as well as applied in correcting sensor problems, observing model uncertainties as a dynamic disturbance, (5) can be rewritten as

\[
\begin{bmatrix}
\Delta R \\
\Delta L_g
\end{bmatrix} = \begin{bmatrix}
\dot{\Delta R} - \omega_L \Delta L_g \\
\Delta L_g \dot{\omega}_L
\end{bmatrix} + \begin{bmatrix}
\Delta L_g \\
\dot{\Delta L}_g
\end{bmatrix} + \begin{bmatrix}
\Delta v_d \\
\Delta v_q
\end{bmatrix}
\] (16)

where \( \Delta R = R_e - R_{go}, \Delta L = L_e - L_{go} \). \( v_d \) and \( v_q \) represent disturbances caused by supplementary unmodeled dynamics and noise interference. Using \( i_{bg} \) and \( i_{gq} \) as the state variables, the state equation of a VSR can be calculated as follows:

\[
\begin{bmatrix}
\frac{di_{bg}}{dt} \\
\frac{di_{gq}}{dt}
\end{bmatrix} = \begin{bmatrix}
A & B \\
C & D
\end{bmatrix} \begin{bmatrix}
i_{bg} \\
i_{gq}
\end{bmatrix} + \begin{bmatrix}
u_{bd} \\
u_{aq}
\end{bmatrix}
\]

The digital implementation of the dynamic in (15) can be represented in the discrete-time domain by approximating the derivative of \( di_{bg} / dt \) and \( di_{gq} / dt \) as follows, where \( T_s \) is the sampling period

\[
\begin{bmatrix}
u_{bg}(k) \\
u_{gq}(k)
\end{bmatrix} = L_{go} C + D \begin{bmatrix}i_{bg}(k) \\
i_{gq}(k)
\end{bmatrix} + \begin{bmatrix}u_{cd}(k) \\
u_{aq}(k)
\end{bmatrix} + \begin{bmatrix}f_d(k) \\
f_q(k)
\end{bmatrix}
\] (18)

where

\[
C = \begin{bmatrix}
i_{bg}(k+1) - i_{bg}(k) \\
i_{gq}(k+1) - i_{gq}(k)
\end{bmatrix},
D = \begin{bmatrix}R_{go} & -\omega_L L_{go} \\
\omega_L L_{go} & R_{go}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{di_{bg}(k)}{dt} \\
\frac{di_{gq}(k)}{dt}
\end{bmatrix} \approx \begin{bmatrix}
i_{bg}(k+1) - i_{bg}(k) \\
i_{gq}(k+1) - i_{gq}(k)
\end{bmatrix}
\] (19)

The future current \( i_{bg}(k+1) \) and \( i_{gq}(k+1) \) can be easily computed using the present sampled values in the following equation

\[
\begin{bmatrix}
i_{bg}(k+1) \\
i_{gq}(k+1)
\end{bmatrix} = \frac{T_s}{L_{go}} E + \begin{bmatrix}R_{go} T_s \\
\omega_L L_{go}
\end{bmatrix} \begin{bmatrix}i_{bg}(k) \\
i_{gq}(k)
\end{bmatrix}
\] (20)

where

\[
E = \begin{bmatrix}u_{bg}(k) - f_d(k) + \omega_L L_{go} i_{bg}(k) - u_{cd}(k) \\
u_{gq}(k) - f_q(k) - \omega_L L_{go} i_{gq}(k) - u_{aq}(k)
\end{bmatrix}
\]

It can be supposed that the values of \( f_d \) and \( f_q \) are constant during each sampling interval [33]-[34]. Under this assumption, the state-space equation can be obtained from (18) as

\[
\begin{bmatrix}
i_{bg}(k+1) \\
i_{gq}(k+1) \\
f_d(k+1) \\
f_q(k+1)
\end{bmatrix} = F \begin{bmatrix}i_{bg}(k) \\
i_{gq}(k) \\
f_d(k) \\
f_q(k)
\end{bmatrix} + \begin{bmatrix}T_s \\
G
\end{bmatrix}
\] (21)

where

\[
F = \begin{bmatrix}A & B \\
C & D
\end{bmatrix},
G = \begin{bmatrix}1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}
\]
Sub-section. The pole placement method, which is to be discussed in the next section, is used to regulate dc side voltage, and to generate appropriate disturbance adaptation using the estimation error. The convergence of the proposed observer can be achieved with an estimated states approach to the actual states. Therefore, the observer is strongly robust and it remains stable for all parameters when \( l_1 \) is varied between 0 and 1, and \( l_2 \) varied between 0 and -40. The observer is stable as long as \( l_1 \) is above 0 and \( l_2 \) above -40. However, severe values of \( l_1 \) decrease the control bandwidth indicating a tradeoff between robustness and speed of response which determines the control gain of the observer. Meanwhile, parameter \( l_2 \) gives a very similar performance in terms of control bandwidth and system robustness.

Fig. 9 illustrates the closed-loop poles of the case, when \( l_1 \) is varied between 0 and 1, and \( l_2 \) =-20. It can be inferred that observer is strongly robust and it remains stable for all parameters when \( l_1 \) is varied between 0 and 1. Moreover, the observer is stable even when the value of \( l_1 \) is slightly smaller than 0 or bigger than 1 and the stable margin is increased.

From (22), the disturbance observer is constructed as shown in (22), and the gains \( (l_1, l_2) \) of the observer are selected by a pole placement method, which is to be discussed in the next sub-section.

From (22), the observer can be implemented in the estimated synchronous reference frame as shown in Fig. 6. Three input values (i.e., rectifier input voltage \( u_{d0}(k) \) and \( u_{q0}(k) \), current \( i_{d0}(k) \) and \( i_{q0}(k) \), grid voltage \( u_{d0}(k) \) and \( u_{q0}(k) \)) are used.

\[
\begin{bmatrix}
    i_{d0}(k+1) \\
    i_{q0}(k+1) \\
    \hat{i}_{d0}(k+1) \\
    \hat{i}_{q0}(k+1)
\end{bmatrix}
= \begin{bmatrix}
    1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & l_1 & 0 \\
    0 & 0 & 0 & l_2
\end{bmatrix}
\begin{bmatrix}
    i_{d0}(k) \\
    i_{q0}(k) \\
    \hat{i}_{d0}(k) \\
    \hat{i}_{q0}(k)
\end{bmatrix}
+ \begin{bmatrix}
    T_{L_{go}} \\
    T_{L_{go}} \\
    T_{L_{go}} \\
    T_{L_{go}}
\end{bmatrix}
\begin{bmatrix}
    g_{d0} \\
    g_{q0} \\
    \ell_{d0} \\
    \ell_{q0}
\end{bmatrix}
+ \begin{bmatrix}
    T_{L_{go}} \\
    T_{L_{go}} \\
    T_{L_{go}} \\
    T_{L_{go}}
\end{bmatrix}
\begin{bmatrix}
    k_{f_{d0}} \\
    k_{f_{q0}} \\
    k_{f_{d0}} \\
    k_{f_{q0}}
\end{bmatrix}
+ \begin{bmatrix}
    1 & 0 & 0 & 0 \\
    1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    \hat{i}_{d0}(k) \\
    \hat{i}_{q0}(k) \\
    \hat{i}_{d0}(k) \\
    \hat{i}_{q0}(k)
\end{bmatrix}
\]

where \( \begin{bmatrix}
    0 & 1 & 0 & 0 \\
    1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & 1 & 0
\end{bmatrix} \).
In order to investigate the impact of parameter mismatch on observer performance, the close-loop poles of observer are studied with variations in the nominal values $L_{go}$. Fig. 11 illustrates the real and imaginary part of the close-loop poles of the observer for $L_{go}$ ranging from 37.5% to 200% of the actual inductance (8 mH). It can be noted from the root locus that the observer is sensitive to $L_{go}$. When the nominal inductance is smaller than the real value, the damping ratio is decreased, also the poles come close to the unit circle and the disturbance observer becomes unstable beyond 37.5% of nominal inductance value. On the other hand, a large value of $L_{go}$ causes the value of the damping ratio increases greatly, reducing speed response of the disturbance observer.
IV. SIMULATION STUDIES

Simulations are carried out to validate the feasibility of the proposed DO-MPC current controller and observer and compare its performance to CF-MPC without the observer when parameter is varied. The control scheme shown in Fig. 7 is simulated in MATLAB/Simulink with parameters given in Table IV.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td>$P$ (kW)</td>
</tr>
<tr>
<td>Grid line-to-line voltage</td>
<td>$E$ (V)</td>
</tr>
<tr>
<td>Grid frequency</td>
<td>$f$ (Hz)</td>
</tr>
<tr>
<td>Equivalent resistance</td>
<td>$R_e$ (Ω)</td>
</tr>
<tr>
<td>Filter inductance</td>
<td>$L_a$ (H)</td>
</tr>
<tr>
<td>dc-link voltage</td>
<td>$u_d$ (V)</td>
</tr>
<tr>
<td>dc-link capacitor</td>
<td>$C$ (μF)</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>$f_s$ (kHz)</td>
</tr>
</tbody>
</table>

The simulation of the system for steps in the amplitude of the input d axis current is shown in Fig. 12. It can be seen that the input currents are definitely sinusoidal and the transient time is very short. The decoupling of the d- and q- axes currents is successfully achieved even during the step transition.

Fig. 13 reports the rectifier input currents in a d-q synchronous reference frame with nominal inductance errors in the DO-MPC strategy. It can be obtained from Fig.13 that the inductance error is set to zero at 0.2 to 0.25 s when the controller operates in normal condition. However, if the nominal inductance is decreased to 4 mH at 0.25 s and increased to 12 mH at 0.3 s, the control performance will be affected. The reactive current appears when the nominal inductance changes.

On the contrary, the proposed DO-MPC strategy shows robust tracking performance without reactive current as is depicted in Fig. 14 (a). Meanwhile, compare with Fig. 13, d-current tracking performance is improved with smaller error. It can be noted in Fig. 14 (b) that the output of the disturbance observer is changed with nominal inductance and the estimated disturbance is almost the same as the actual system disturbance. Thus the estimated value can revise the error of current predictive model through feed forward action and eliminate disturbances.

In order to verify the disturbance observer, a test on comparing the actual system disturbance and the estimated disturbance is implement when nominal inductance variations. It can be seen from Fig. 15, the variation of nominal inductance value $L_{go}$ leads to transient disturbance on d-axis and steady-state disturbance on q-axis of control system. The output value $f_d$ of the observer can track the actual transient disturbance precisely at the moment of $L_{go}$ variation. Meanwhile, the output value $f_q$ changes with the variation of $L_{go}$, which is basically the same as the steady-state disturbance on q-axis caused by change of $L_{go}$. Thus the system disturbance can be effectively observed by the disturbance observer which shows a good performance.
Fig. 14. Simulation results with nominal inductance variations in the proposed DO-MPC strategy. (a) Input d- and q-currents. (b) Actual system disturbance and estimated results.

Fig. 15. Actual system disturbance compared with estimated disturbance when nominal inductance variations.

V. EXPERIMENTAL VALIDATIONS

To evaluate the performance of the robust model predictive current control based on a disturbance observer of VSR, experimental studies are performed on a 1.1 kW VSR prototype in a fully digital signal processor (DSP) system based on TMS320F28335. It consists of a three-phase insulated-gate bipolar transistors (IGBTs) based rectifier with anti-paralleling diodes and six single channel drivers, which is the so-called VSR. Three Hall-effect current and voltage sensors are employed to measure three input currents, the dc-link voltage and two line to line power-source voltages, respectively. The analog signals are first processed with a Butterworth low-pass filter and then sampled at the frequency of 5 kHz. The VSR is connected to power-source via a 9.9 kVA three-phase transformer and an 8 mH filter inductor. The system parameters are listed in Table IV.

A. Steady-State Response Tests

Fig. 16 shows the steady-state test result when the reference current is 7.5 A peak with unity power factor. The test result demonstrates the excellent steady state response of the proposed DO-MPC algorithm. The rectifier input current is highly sinusoidal with a measured total harmonic distortion (THD) of 1.93%. Compared with the variable switching frequency MPC algorithm [35], the output of DO-MPC is first passed through modulator which determines the switch positions. Therefore, small current harmonic is obtained under low switching frequency (5 kHz). The sum of duration time of three chosen voltage vectors in Fig. 16(c) is one switching period $T_s$. 

![Graph](image-url)
Fig. 16 Steady state test result of DO-MPC. (a) The voltage and current waveforms. (b) Harmonic spectrum of phase-a current. (c) The waveforms of the first active vector and duration time of three chosen voltage vectors.

B. Dynamic Response Tests

An important aspect of any control system is its dynamic response to reference changes. Fig. 17 depicts the system behavior for a current reference step from 6- to 9- A peak and vice versa when the rectifier was operating in steady states. It can be seen that the transient time is very short (less than 2 ms), indicating the excellent dynamic response of this advanced current controller.

There exists small distortion in the current under transient state process. This is attributed to the absence of voltage loop and instability of dc-link voltage causes the pulsation of reference current. Furthermore, sampling, computation and output of the controller should be completed at the same time in ideal condition. However, time delay is existed in real control system. The MPC algorithm with control delay compensation can obtain more satisfied current [36]-[37].

Fig. 18 illustrates the current and dc-link voltage with the load stepped from 0.5 kW to 1 kW and vice versa.

Fig. 18 Current and dc-link voltage behavior during load step with DO-MPC algorithm. (a) Step change from 0.5 kW to 1 kW. (b) Step change from 1 kW to 0.5 kW.

As can be seen from the Fig. 18, the transition process of the current is stable and definitely sinusoidal. At the instant of the load step, the dc-link voltage $u_{\text{dc}}$ slightly decreases with value of 8 V due to increase in the output power while $u_{\text{dc}}$ slightly increases with value of 6 V due to decrease in the output power. The proposed control algorithm manages to keep the dc-link voltage at the desired values.

The above experiment results, as is shown in sub-section A and B, validate the excellent operation of DO-MPC algorithm under normal condition.

C. Investigation of Model Mismatch

Fig.19 compares the performance of VSR system adopting
CF-MPC with that employing DO-MPC when there are ±50% errors in the inductance parameter.

![Experimental waveforms with nominal inductance variations adopting CF-MPC and the proposed DO-MPC (a) inductance parameter decreased by 50%. (b) inductance parameter increased by 50%](image1)

**Fig.19** Experimental waveforms with nominal inductance variations adopting CF-MPC and the proposed DO-MPC (a) inductance parameter decreased by 50%. (b) inductance parameter increased by 50%

CF-MPC without observer is firstly applied in the experiment, and then DO-MPC with observer is adopted. Inaccurate predictive reference voltage is obtained by CF-MPC due to model mismatch, as shown in Fig.19. Thus, error occurs in the control of $i_{gd}$, $i_{gq}$ and the controller can not track the reference current exactly. When DO-MPC is adopted, the observer output modifies the predictive model via feed-forward compensation and eliminates the current error. Thus the current $i_{gd}$ and $i_{gq}$ tracks the reference value quickly and accurately, and the robustness control of MPC is realized.

In order to test the output compensation of the observer, the observer output is compared with the real system error, as depicted in Fig.20. Although system error is existed with CF-MPC algorithm, the observer output is zero as no observer is used. When DO-MPC algorithm is applied, the observer output $f_d$ and $f_q$ estimate the system error and the oscillation process is basically the same with the system error. In this way, the observer estimates the system error caused by model mismatch effectively.

![Actual system disturbance compared with estimated disturbance when nominal inductance variations (a) inductance parameter decreased by 50% (b) inductance parameter increased by 50%](image2)

**Fig.20** Actual system disturbance compared with estimated disturbance when nominal inductance variations (a) inductance parameter decreased by 50% (b) inductance parameter increased by 50%

The change of the observer output is estimated via gradual change of inductance parameter in the experiment, as shown in Fig.21. The output of the observer in q-axis $f_q$ changes with the gradual change of $L_{go}$. At the instant of $L_{go}$ variation, transient fluctuation appears in the output of the observer in d-axis $f_d$ Thus the observer can adjust the compensation adaptively according to the change of $L_{go}$, and it has great agreement with the simulation result.

![Observer output comparison with gradual change of inductance parameter](image3)

**Fig.21** Observer output comparison with gradual change of inductance parameter
shown that the reference tracking time is short, resulting in an excellent dynamic response. The output of the observer in q-axis $f_q$ changes with the step of d-axis current $i_{gd}$ and it has great agreement with the proposed observer theoretical analysis.

VI. CONCLUSION

A fully digital control of three-phase voltage source PWM rectifiers is proposed in this paper and a Luenberger disturbance observer is adopted for its estimation. A constant switching frequency algorithm of MPC current control has been applied considering its simple implementation, fast dynamic response, easy EMI filter design and low sampling frequency requirement. An adaptive internal model for estimating uncertainty dynamics has been developed and included within the voltage feedback structure. Combining with disturbance observer, the developed DO-MPC method shows excellent performance even when a large parameter mismatch occurs. The observer has excellent robustness with nominal inductance $L_{go}$ varied from 37.5% to 200% of the actual value. The proposed algorithm is verified by simulation and experimental tests on a VSR. Compared to the CF-MPC without disturbance observer, the presented controller provides robust performance considering the parameter errors in the inductor filter and the reactive current can be eliminated in case of nominal inductance error ranging from −50% to +50% of the actual inductance.

REFERENCES


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